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Review of the Doctoral Thesis

The Boundary Element Method for Shape Optimization in 3D

by Ing. Jan Zapletal

An accurate and efficient solution of shape optimization problems is of an emerging interest in engineering and industrial applications, and covers many challenges from a mathematical point of view. Main ingredients in shape optimization are the definition of appropriate cost functionals, the minimization of the cost functional, e.g. by using gradient based methods, the solution of the primal and adjoint boundary value problems, and the description and update of the underlying boundary description.

Although the topic of shape optimization is well established in the scientific community, the combination of state of the art direct solution methods for the primal and adjoint boundary value problems with subdivision algorithms from computer graphics is by no means a trivial task. The aim of this thesis is to contribute to an efficient solution of the Bernoulli free boundary problem by using boundary element methods and subdivision algorithms.

The thesis consists of 4 chapters, and an impressive bibliography with 143 references.

The Introduction (3 pages) gives a clear overview on the topics and content of the thesis.

Chapter 2 (38 pages) on the Bernoulli free boundary prolem summarizes the required preliminaries on the formulation of the problem, the used function spaces, the existence of optimal domains and gradient type minimization methods, and boundary integral equation methods for the solution of the primal and adjoint boundary value problems. Instead of $\Omega \subset \mathbb{R}^d$, see page 5, it would be sufficient to consider $\Omega \subset \mathbb{R}^3$ only. Note that d is not given at all. The partial derivatives as used in (2.3), page 7, are weak derivatives, and not in the distributional sense. The latter are used to define Sobolev spaces $H^s(\Omega)$ by means of the Fourier transform in $\mathbb{R}^3$ and restriction to Ω. Indeed, under reasonable conditions, e.g. the cone condition, it turns out that the

Sobolev spaces $W^{s,2}(\Omega)$ and $H^s(\Omega)$ coincide; the notation for convenience as used on page 7 is therefore not correct. In Subsection 2.2 several choices of possible cost functionals are discussed, of particular interest is the tracking in $H^{-1/2}(\Gamma)$ where the norm can either be realized by using a boundary integral operator, or the Dirichlet energy norm (2.25) which corresponds to the Neumann to Dirichlet map, and which can be realized by solving a transmission problem. Note that the Neumann datum can be obtained rather easily when solving the Dirichlet problem in Ω and using Lagrange multipliers, see page 16. In the remainder of this subsection a careful review of the shape calculus is given, and the use of boundary integral equation methods is discussed.

Chapter 3 (16 pages) summarizes different representations of smooth curves and surfaces. This a very helpful overview on existing approaches, as they will be used for the shape optimization algorithm.

Chapter 4 (54 pages) represents the main part of the thesis, in particular the combination of boundary element methods and their efficient implementation on high performance computer architectures with shape optimization algorithms and industrial applications. Unfortunately, the definition of the global mesh size h as given on page 59 is not correct, instead one has to use

$$h := \max_k \sqrt{\Delta_k}\,.$$

While the boundary element error estimates (4.12) for the Dirichlet problem and (4.21) for the Neumann problem are cited correctly, i.e. implying linear and quadratic convergence, respectively, the estimated order of convergence (eoc) as given in Table 4.1, page 76, does not fit. Moreover, the mapping properties of the modified hypersingular integral operator $\widetilde{D}$ are not correct. However, besides of these inconsistencies, the content of this chapter include the main results of this thesis, in particular a very detailed discussion of the efficient boundary element implementation, and the application on shape optimization problems including an industrial example. These results are quite impressive.

The thesis closes with some conclusions and comments on further research.

Without any doubt, the doctoral thesis fulfils the stipulated goal, and Jan Zapletal shows his ability to work in the challenging interdisciplinary fields of applied mathematics, scientific computing, and engineering applications. The thesis is written in an excellent form. The summary of publications includes 5 journal papers which is quite impressive for a PhD student. Without any doubt, I strongly support the acceptance of the PhD thesis.

From the point of scientific computing I find the thesis excellent since J. Zapletal showed his ability to formulate and implement state of the art mathematical algorithms on state of the art computing facitilies. Unfortunately, from the point of numerical analysis, this picture is not so clear due to some mathematical inconsistencies as mentioned in the report. Howver, I still rate the thesis to be **very good**.

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